GOAL ORIENTED ERROR ESTIMATION

for the Element Free Galerkin method

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Summary

A novel approach for implicit residual-type error estimation in mesh-free methods is presented. This allows to compute upper and lower bounds of the error in energy norm with the ultimate goal of obtaining bounds for outputs of interest. The proposed approach precludes the main drawbacks of standard residual type estimators circumventing the need of flux-equilibration and resulting in a simple implementation that avoids integrals on edges/sides of a domain decomposition (mesh). This is especially interesting for mesh-free methods.

State of the art

- **Liu, Jun, Sihling, Chen, Hao, 1997 → Wavelet solution as error indicator**
- **H.J. Chung, T. Belytschko, Comp. Mechanics, 1998 → Stress projection**
- **L. Gavete, J.L. Cuesta, IJNME, 2002 → Recovery based**
- **...**

Implicit residual based estimators have not been proposed for mesh-free methods ! (Standard in FEM)

Assessment of bounds and functional outputs of interest are still an open topic in mesh-free methods

State of the art

Implicit residual based error estimators in FE:

- **Solve local problems with Dirichlet b.c. (Lower bound)**
- **Solve local problems with Neumann b.c. (Upper bound):**
  - **Equilibrated hybrid fluxes [Ladevèze; Ainsworth,...]**
  - **Flux-free [Carstensen & Funken; Machiels, Maday & Patera; Morin, Nochetto & Siebert; Parés, Díez & Huerta]**

Here we present an implicit residual-type flux-free error estimator for the EFG method

GOALS:

- **Assessment of bounds**
- **Outputs of interest**

Error equations

$$a(e_h, v) = R^P(v) \quad \forall v \in \mathcal{V}^h$$ COMPUTATIONALLY UNAFFORDABLE

$$u_h \neq u_H + e_h \text{ as } \mathcal{V}^H \not\subset \mathcal{V}^h$$ DIFFERENT TO FEM

$$l^{\mathcal{O}}(e_h) = a(e_h, \epsilon_h) + a(e_h, \psi_H)$$

= 0 FEM

≠ 0 EFG

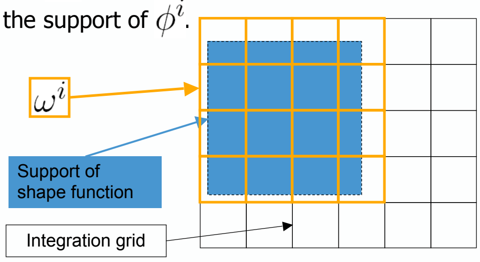
Theorem: $|a(e_h, \psi_H) - a(\tilde{e}, \psi_H)| \leq C\, h^p H^p$

Local decomposition

Main idea: the EFG basis $\{\phi^i\}_{i=1\ldots n_{np}}$ is a partition of unity, i.e.

$$\sum_{i=1}^{n_{np}} \phi^i = 1 \implies R^P(v) = \sum_{i=1}^{n_{np}} R^P(\phi^i v) \quad \forall v \in \mathcal{V}^h.$$

Let ω^i denote the smallest integration sub-grid that includes the support of ϕ^i.



We will solve a local problem in each ω^i.

Note that ϕ^i is zero out of ω^i.

For $i = 1, \ldots, n_{np}$, compute $\tilde{e}^{\omega^i} \in \mathcal{V}^h_{\omega^i}$ s.t.

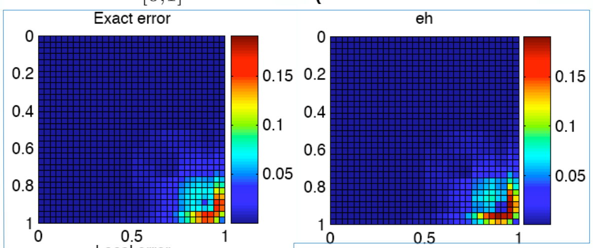
$$a_{\omega^i}(\tilde{e}^{\omega^i}, v) = R^P(\phi^i v) \quad \forall v \in \mathcal{V}^h_{\omega^i}.$$

Define the global estimate, $\tilde{e} \in \mathcal{V}^h_{\text{brok}}$

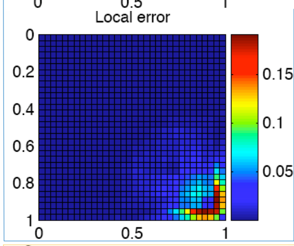
$$\tilde{e} := \sum_{i=1}^{n_{np}} \tilde{e}^{\omega^i}.$$

Example

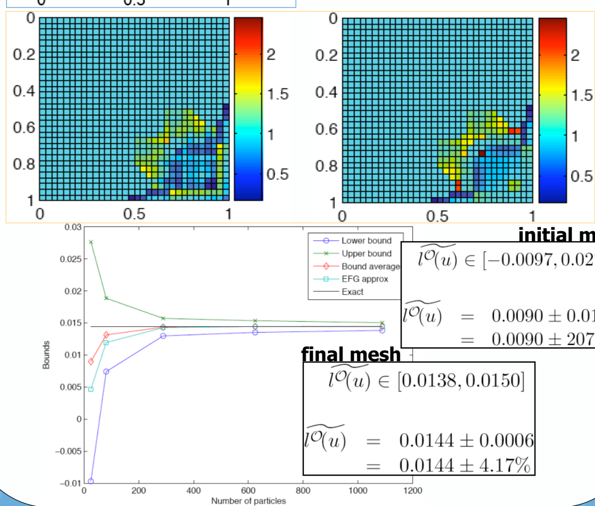
$$\ell^{\mathcal{O}}(u) = \int_{[0,1]^2} u \, d\Omega \qquad \begin{cases} -\Delta u = f & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases}$$



Energy norm of error and estimate



Local effectivity index with respect exact error (left) and reference error (right)



initial mesh

$$\widetilde{l^{\mathcal{O}}(u)} \in [-0.0097, 0.0276]$$

$$\widetilde{l^{\mathcal{O}}(u)} = 0.0090 \pm 0.0187 = 0.0090 \pm 207.78\%$$

final mesh

$$\widetilde{l^{\mathcal{O}}(u)} \in [0.0138, 0.0150]$$

$$\widetilde{l^{\mathcal{O}}(u)} = 0.0144 \pm 0.0006 = 0.0144 \pm 4.17\%$$

Ref.

[1] Y. Vidal and A. Huerta. Goal oriented error estimation for the Element Free Galerkin. *Lecture Notes in Computational Science and Engineering, in press.*